NAME:_____ PD:_____ DATE:_____

State the Degree, Leading Coefficient, and End Behavior of each function.

1. $h(x) = 4x^3 - 6x + 1$

D: 3 $4x^3$ ↙ ↗
LC: 4

$x \to +\infty, f(x) \to \infty$

$x \to -\infty, f(x) \to -\infty$

2. $f(x) = -3x^4 + 5x^3 + 2x - 4$

D: 4 $-3x^4$ ↓ ↓
LC: -3

$X \to +\infty, f(x) \to -\infty$

$X \to -\infty, f(x) \to -\infty$

3. $m(x) = -\frac{1}{4}x^5 + 7$

D: 5 $-\frac{1}{4}x^5$ ↖ ↘
LC: $-\frac{1}{4}$

$x \to +\infty, f(x) \to -\infty$

$x \to -\infty, f(x) \to \infty$

Factor each sum and difference of cubes.

4. $3x^3 - 81$

$3(x^3 - 27)$ $a = x$ $b = 3$

$3(x-3)(x^2+3x+9)$

5. $2x^6 + 54$

$2(x^6 + 27)$ $a = x^2$ $b = 3$

$2(x^2+3)(x^4-3x^2+9)$

6. $8x^3 - 125$ $a = 2x$ $b = 5$

$(2x-5)(4x^2+10x+25)$

List the possible rational roots (p/q's) and then find all the real roots of each polynomial equation.

7. $x^3 - 2x^2 - 19x + 20 = 0$

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$-4 \mid 1 \quad -2 \quad -19 \quad 20$
$\quad\quad\downarrow \quad -4 \quad 24 \quad -20$
$\quad\quad 1 \quad -6 \quad 5 \quad 0$

$x^2 - 6x + 5 = 0$
$(x-5)(x-1) = 0$

$x = 5, 1, -4$

8. $x^3 - 6x^2 + 11x - 6 = 0$

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6$

$1 \mid 1 \quad -6 \quad 11 \quad -6$
$\quad\quad\downarrow \quad 1 \quad -5 \quad 6$
$\quad\quad 1 \quad -5 \quad 6 \quad 0$

$x^2 - 5x + 6 = 0$
$(x-2)(x-3) = 0$

$x = 2, 3, 1$

9. $10x^4 - 13x^3 - 21x^2 + 10x + 8 = 0$

$\frac{p}{q} = \pm 1, \pm\frac{1}{2}, \pm\frac{1}{5}, \pm\frac{1}{10}, \pm 2, \pm\frac{2}{5}, \pm 4, \pm\frac{4}{5}, \pm 8, \pm\frac{8}{5}$

$-1 \mid 10 \quad -13 \quad -21 \quad 10 \quad 8$
$\quad\quad\downarrow \quad -10 \quad 23 \quad -2 \quad -8$
$\quad\quad 10 \quad -23 \quad 2 \quad 8 \quad 0$

$2 \mid 10 \quad -23 \quad 2 \quad 8$
$\quad\quad\downarrow \quad 20 \quad -6 \quad -8$
$\quad\quad 10 \quad -3 \quad -4 \quad 0$

$10x^2 - 3x - 4 = 0$
$(x - \frac{8}{10})(x + \frac{5}{10}) = 0$

$x = \frac{4}{5}, -\frac{1}{2}, 2, -1$

Write a polynomial equation in standard form given the zeros.

10. $2, -1, \frac{1}{3}$

$(x-2)(x+1)(3x-1) = 0$
$(x^2 + x - 2x - 2)(3x-1) = 0$
$(x^2 - x - 2)(3x-1) = 0$
$3x^3 - x^2 - 3x^2 + x - 6x + 2 = 0$

$3x^3 - 4x^2 - 5x + 2 = 0$

11. $-2i, 2i, 0, -4$

$x(x+2i)(x-2i)(x+4) = 0$
$x(x^2 - 4i^2)(x+4) = 0$
$x(x^2+4)(x+4) = 0$
$(x^3 + 4x)(x+4) = 0$

$x^4 + 4x^3 + 4x^2 + 16x = 0$

12. $2 + i, 2 - i, 5$

$x = \frac{4}{5}, -\frac{1}{2}, 2, -1$

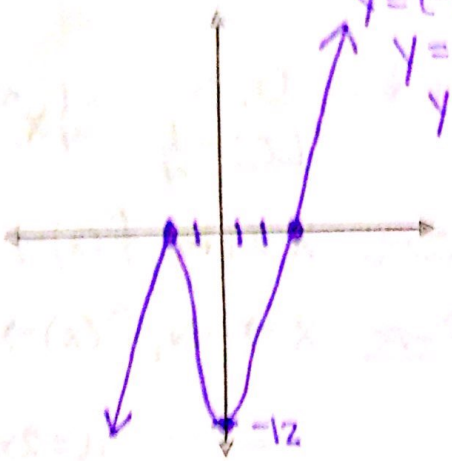
$(x - (2+i))(x - (2-i))(x-5) = 0$
$(x - 2 - i)(x - 2 + i)(x-5) = 0$
$(x^2 - 2x + xi - 2x + 4 - 2i - xi + 2i - i^2)(x-5) = 0$
$(x^2 - 4x + 4 - i^2)(x-5) = 0$
$(x^2 - 4x + 5)(x-5) = 0$

$x^3 - 9x^2 + 25x - 25 = 0$

Graph each polynomial function showing zeros, y-intercept, and end behavior. Identify the characteristics of each function.

13. $f(x) = (x-3)(x+2)^2$

$y = (0-3)(0+2)^2$
$y = (-3)(2)^2$
$y = -3(4)$
$y = -12$



Zeros	Multiplicity	Cross/Bounce?
3	1	Cross
-2	2	Bounce

Y-intercept: (0, -12)

Degree of the polynomial: 3

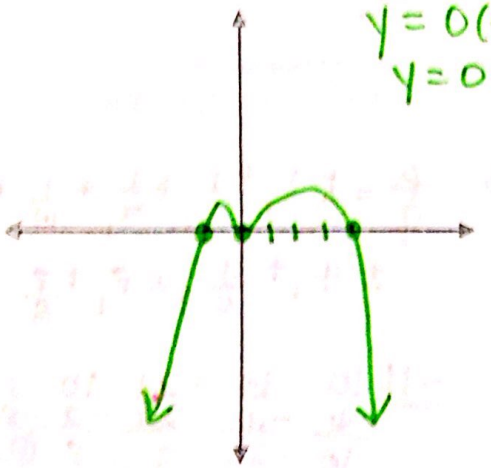
Pos./Neg. Leading Coefficient? positive

End Behavior: $x \to \infty$, $f(x) \to \infty$

$x \to -\infty$, $f(x) \to -\infty$

14. $f(x) = -x^2(x-4)(x+1)$

$y = -(0)^2(0-4)(0+1)$
$y = 0(-4)(1)$
$y = 0$



Zeros	Multiplicity	Cross/Bounce?
0	2	Bounce
4	1	Cross
-1	1	Cross

Y-intercept: (0, 0)

Degree of the polynomial: 4

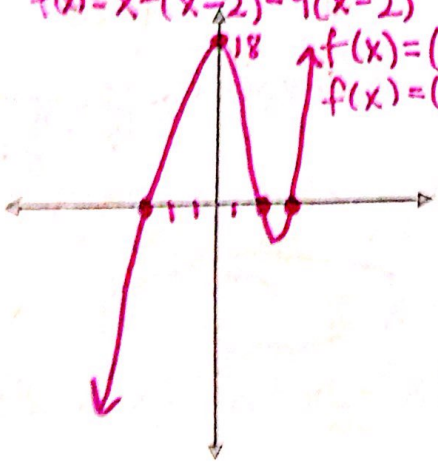
Pos./Neg. Leading Coefficient? Negative

End Behavior: $x \to +\infty$, $f(x) \to -\infty$

$x \to -\infty$, $f(x) \to -\infty$

15. $f(x) = (x^3 - 2x^2 - 9x + 18)$

$f(x) = x^2(x-2) - 9(x-2)$
$f(x) = (x^2-9)(x-2)$
$f(x) = (x+3)(x-3)(x-2)$



Zeros	Multiplicity	Cross/Bounce?
-3	1	Cross
3	1	Cross
2	1	Cross

Y-intercept: (0, 18)

Degree of the polynomial: 3

Pos./Neg. Leading Coefficient? Positive

End Behavior: $x \to \infty$, $f(x) \to \infty$

$x \to -\infty$, $f(x) \to -\infty$

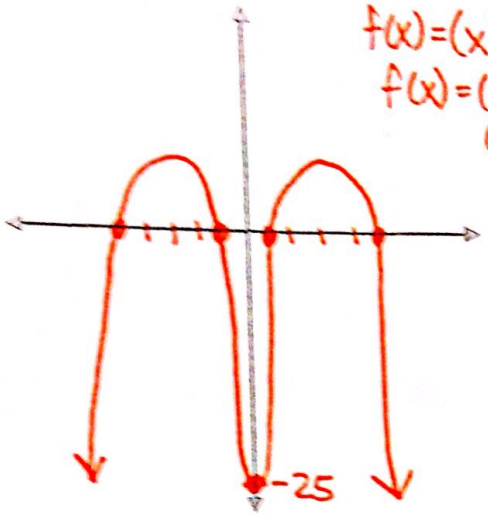
16. $f(x) = -x^4 + 26x^2 - 25$

25 / 25 / 1 / 25

$f(x) = (x^2+25)(x^2+1)$

$f(x) = (x^2-25)(x^2+1)$

$f(x) = (x+5)(x-5)(x+1)(x-1)$



Zeros	Multiplicity	Cross/Bounce?
-5	1	Cross
5	1	Cross
-1	1	Cross
1	1	Cross

Y-intercept: (0, -25)

Degree of the polynomial: 4

Pos./Neg. Leading Coefficient? Negative

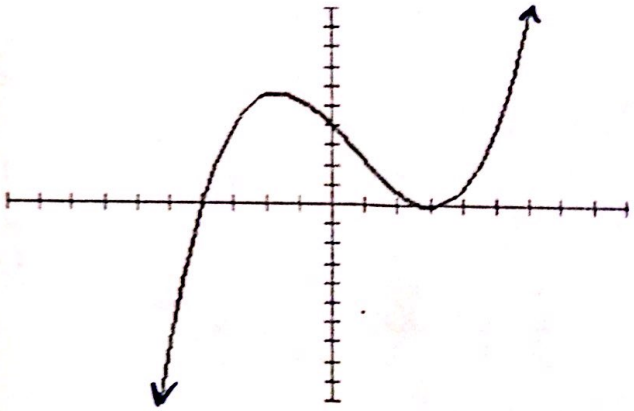
End Behavior: $x \to \infty$, $f(x) \to -\infty$

$x \to -\infty$, $f(x) \to -\infty$

Use the graph at the right to answer questions 17 – 23.

17. In the table below, state the zeros and what their **LEAST** multiplicity could be.

Zeros	-4	3
Multiplicity	1	2



18. Is the leading coefficient of the function positive or negative?

Positive

19. Is the degree of the function even or odd?

Odd

20. How many turning points does the function have?

2

21. What is the **LEAST** degree this polynomial could be?

3rd

22. Is the graph increasing or decreasing at the interval $(3, \infty)$?

Increasing

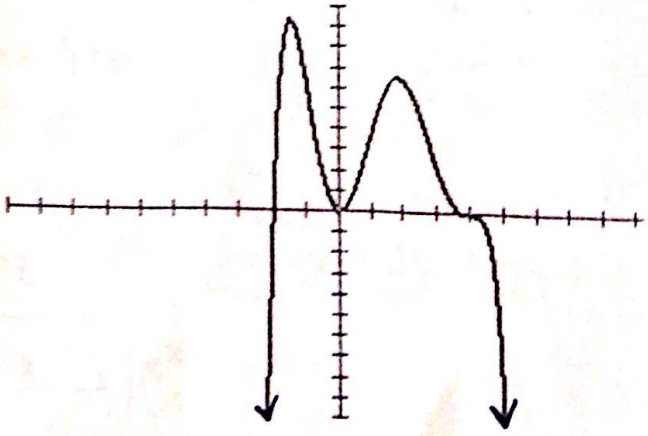
23. How many **absolute max/min** points does this graph have?

None

Use the graph at the right to answer questions 24 – 29.

24. In the table below, state the zeros and what their **LEAST** multiplicity could be.

Zeros	-2	0	4
Multiplicity	1	2	3



25. Is the leading coefficient of the function positive or negative?

Negative

26. Is the degree of the function even or odd?

Even

27. How many turning points does the function have?

5

28. What is the **LEAST** degree this polynomial could be?

6th

29. How many **relative max/min points** does this graph have?

2 / 1