

Unit 3 Quiz 1 Review

Factor the sum and difference of cubes.

1. $x^3 + 27$ $a=x$ $b=3$

$$(x+3)(x^2 - (x)(3) + (3)^2)$$

$$(x+3)(x^2 - 3x + 9)$$

2. $64y^3 - 27$

$a=4y$
 $b=3$

$$(4y-3)((4y)^2 + (4y)(3) + (3)^2)$$

$$(4y-3)(16y^2 + 12y + 9)$$

3. $32x^6 - 500x^3$

$a=2x$
 $b=5$

$$4x^3(8x^3 - 125)$$

$$4x^3(2x-5)((2x)^2 - (2x)(5) + 5^2)$$

$$4x^3(2x-5)(4x^2 - 10x + 25)$$

4. $f(x) = x^3 + 3x^2 - 6x - 8$

A. What are all of the possible rational zeros of $f(x)$?

$$\frac{p}{q} = \pm \frac{1, 2, 4, 8}{1} = \boxed{\pm 1, \pm 2, \pm 4, \pm 8}$$

B. Determine all of the rational roots. Show work.

$$\begin{array}{r} 2 \overline{) 1 \ 3 \ -6 \ -8} \\ \underline{-2 \ 10 \ 8} \\ 1 \ 5 \ 4 \ 0 \end{array}$$

$$x^2 + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

$$\begin{array}{r} 4 \\ \times 1 \\ \hline 4 \end{array}$$

$$\boxed{x = -4, -1, 2}$$

Solve each.

5. $x^2 - 4x + 4 = 0$

$$(x-2)(x-2) = 0$$

$$\boxed{x = 2 \text{ (Mult. 2)}}$$

6. $x^2 - 81 = 0$

$$(x+9)(x-9) = 0$$

$$\boxed{x = -9, 9}$$

7. $x^2 + x - 12 = 0$

$$(x+4)(x-3) = 0$$

$$\boxed{x = -4, 3}$$

8. $(x^3 - 5x^2) - (4x + 20) = 0$

$$x^2(x-5) - 4(x+5) = 0$$

$$(x-5)(x^2 - 4) = 0$$

$$(x-5)(x+2)(x-2) = 0$$

$$\boxed{x = 5, -2, 2}$$

9. $2x^2 + 13x - 24 = 0$

$$(2x^2 + 16x) - (3x - 24) = 0$$

$$2x(x+8) - 3(x-8) = 0$$

$$(x+8)(2x-3) = 0$$

$$\boxed{x = -8, \frac{3}{2}}$$

10. $6x^2 - 31x + 5 = 0$

$$(6x^2 - 30x) - (x+5) = 0$$

$$6x(x-5) - 1(x-5) = 0$$

$$(x-5)(6x-1) = 0$$

$$\boxed{x = 5, \frac{1}{6}}$$

11. $(4x + 12) + (x^2 + 3x) = 0$

$$4(x+3) + x(x+3) = 0$$

$$(x+3)(4+x) = 0$$

$$\boxed{x = -3, -4}$$

12. $(x^3 + 5x^2) - (9x - 45) = 0$

$$x^2(x+5) - 9(x-5) = 0$$

$$(x+5)(x^2 - 9) = 0$$

$$(x+5)(x+3)(x-3) = 0$$

$$\boxed{x = -5, -3, 3}$$

13. $x^2 - 8x - 48 = 0$

$$(x-12)(x+4) = 0$$

$$\boxed{x = 12, -4}$$

Multiple Choice Select the best choice for each problem.

14. Find all of the rational zeros of $g(x) = 2x^3 + 4x^2 - 2x - 4$. *Look at your table in your calculator*

- A) $x = \{2, 4, -2, -4\}$ B) $x = \{-2, 1, 2\}$ C) $x = \{4, 1, -4\}$
 D) $x = \{-4, -1, 4\}$ **(E) $x = \{-2, -1, 1\}$**

15. Consider $g(x) = 2x^3 + 4x^2 - 2x - 8$.

If we were to list all of the possible rational zeros using the rational root theorem (p/q) for the polynomial we would have how many possible rational zeros?

$$\frac{P}{q} = \pm \frac{1, 2, 4, 8}{1, 2} = \pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 8$$

- A) 3 B) 4 C) 5 D) 8 **(E) more than 8 * 10 possible ***

Find all zeros and list any multiplicity.

16. $y = x^3(x-2)(x+4)^2$

$$\begin{aligned} x &= 0 \text{ (mult. 3)} \\ x &= 2 \\ x &= -4 \text{ (mult. 2)} \end{aligned}$$

17. $y = x(x+2)^2(x+3)$

$$\begin{aligned} x &= 0 \\ x &= -2 \text{ (mult. 2)} \\ x &= -3 \end{aligned}$$

18. $y = x^4 + 7x^3 + 12x^2$

$$\begin{aligned} y &= x^2(x^2 + 7x + 12) \\ y &= x^2(x+4)(x+3) \\ x &= 0 \text{ (Mult. 2)}, \\ & -4, -3 \end{aligned}$$

Find all the zeros of the function using synthetic division, factoring or quadratic formula.

Find p/q 's.

19. $f(x) = 2x^3 + 3x^2 - 59x - 30$

$$\frac{P}{q} = \pm \frac{1, 2, 3, 5, 6, 10, 15, 30}{1, 2}$$

$$\frac{P}{q} = \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2}, \pm 6, \pm 10, \pm 15, \pm \frac{15}{2}, \pm 30$$

$$\begin{array}{r|rrrr} -6 & 2 & 3 & -59 & -30 \\ & \downarrow & -12 & 54 & 30 \\ \hline & 2 & -9 & -5 & 0 \end{array}$$

~~$\frac{-10}{-10} \frac{1}{1}$~~

$$\begin{aligned} 2x^2 - 9x - 5 &= 0 \\ (2x^2 - 10x) + (x - 5) &= 0 \end{aligned}$$

$$\begin{aligned} 2x(x-5) + 1(x-5) &= 0 \\ (x-5)(2x+1) &= 0 \\ x &= 5, -\frac{1}{2}, -6 \end{aligned}$$

21. $f(x) = 10x^4 - 13x^3 - 21x^2 + 10x + 8$
 $\frac{P}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm 2, \pm \frac{2}{5}, \pm 4, \pm \frac{4}{5}, \pm 8, \pm \frac{8}{5}$

$$\frac{P}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm 2, \pm \frac{2}{5}, \pm 4, \pm \frac{4}{5}, \pm 8, \pm \frac{8}{5}$$

$$\begin{array}{r|rrrrr} -1 & 10 & -13 & -21 & 10 & 8 \\ & \downarrow & -10 & 23 & -2 & -8 \\ \hline & 10 & -23 & 2 & 8 & 0 \end{array}$$

$$10x^3 - 23x^2 + 2x + 8 = 0$$

20. $f(x) = x^3 - 5x^2 - 48x + 108$

$$\frac{P}{q} = \pm 1, \pm 2, \pm 108, \pm 54, \pm 3, \pm 36, \pm 4, \pm 27, \pm 6, \pm 18, \pm 9, \pm 12$$

$$\begin{array}{r|rrrr} -6 & 1 & -5 & -48 & 108 \\ & \downarrow & -6 & 66 & -108 \\ \hline & 1 & -11 & 18 & 0 \end{array}$$

$$\begin{aligned} x^2 - 11x + 18 &= 0 \\ (x-9)(x-2) &= 0 \end{aligned}$$

$$x = 9, 2, -6$$

~~$\frac{18}{-11} \frac{2}{-11}$~~

$$\begin{array}{r|rrrr} 2 & 10 & -23 & 2 & 8 \\ & \downarrow & 20 & -6 & -8 \\ \hline & 10 & -3 & -4 & 0 \end{array}$$

$$\begin{aligned} 10x^2 - 3x - 4 &= 0 \\ (10x^2 - 8x) + (5x - 4) &= 0 \end{aligned}$$

$$\begin{aligned} 2x(5x-4) + 1(5x-4) &= 0 \\ (5x-4)(2x+1) &= 0 \end{aligned}$$

$$x = \frac{4}{5}, -\frac{1}{2}, 2, -1$$