

Unit 2B Study Guide

Perform the following operations on functions and give the domain.

$$f(x) = x^2$$

$$g(x) = 2x - 7$$

$$h(x) = -3x^2 + 4$$

1) $(f+g)(x)$

$$f(x) + g(x)$$

$$\boxed{x^2 + 2x - 7}$$

$$\boxed{D: \text{All } \mathbb{R}'s}$$

2) $(f-g)(x)$

$$f(x) - g(x)$$

$$x^2 - (2x - 7)$$

$$\boxed{x^2 - 2x + 7}$$

$$\boxed{D: \text{All } \mathbb{R}'s}$$

3) $g(x) * f(x)$

$$(2x-7)(x^2)$$

$$\boxed{2x^3 - 7x^2}$$

$$\boxed{D: \text{All } \mathbb{R}'s}$$

4) $\left(\frac{f}{g}\right)(x)$

$$\frac{f(x)}{g(x)}$$

$$\frac{x^2}{2x-7}$$

$$\boxed{D: \text{All } \mathbb{R}'s \text{ except } x \neq \frac{7}{2}}$$

5) $f(g(x))$

$$x^2$$

$$(2x-7)^2$$

$$(2x-7)(2x-7)$$

$$4x^2 - 14x - 14x + 49$$

$$\boxed{4x^2 - 28x + 49}$$

$$\boxed{D: \text{All } \mathbb{R}'s}$$

6) $g(f(x))$

$$2x-7$$

$$2(x^2) - 7$$

$$\boxed{2x^2 - 7}$$

$$\boxed{D: \text{All } \mathbb{R}'s}$$

7) $f(g(2))$

$$x^2$$

$$(2x-7)^2$$

$$(2(2)-7)^2$$

$$(4-7)^2$$

$$(-3)^2 = \boxed{9}$$

8) $h(h(-4))$

$$-3x^2 + 4$$

$$-3(-3x^2 + 4)^2 + 4$$

$$-3(-3(-4)^2 + 4)^2 + 4$$

$$-3(-3(16) + 4)^2 + 4$$

$$-3(-48 + 4)^2 + 4$$

$$-3(-44)^2 + 4$$

$$-3(1936) + 4$$

$$-5808 + 4$$

$$\boxed{-5804}$$

9) $f(g(h(3)))$

$$x^2$$

$$(2x-7)^2$$

$$(2(-3x^2+4)-7)^2$$

$$(2(-3(3)^2+4)-7)^2$$

$$(2(-3(9)+4)-7)^2$$

$$(2(-27+4)-7)^2$$

$$(2(-23)-7)^2$$

$$(-46-7)^2$$

$$\boxed{(-53)^2}$$

$$\boxed{2809}$$

Find the inverse of the $f(x)$.

10) $f(x) = x^2 - 4$

$$y = x^2 - 4$$

$$x = y^2 - 4$$

$$+4 \quad +4$$

$$\sqrt{x+4} = \sqrt{y^2}$$

$$\pm\sqrt{x+4} = y$$

$$\boxed{f^{-1}(x) = \pm\sqrt{x+4}}$$

11) $f(x) = (x-2)^2 + 3$

$$y = (x-2)^2 + 3$$

$$x = (y-2)^2 + 3$$

$$-3 \quad -3$$

$$\sqrt{x-3} = \sqrt{(y-2)^2}$$

$$\pm\sqrt{x-3} = y-2$$

$$\pm\sqrt{x-3} + 2 = y$$

12) $f(x) = x^3 - 4$

$$y = x^3 - 4$$

$$x = y^3 - 4$$

$$+4 \quad +4$$

$$\sqrt[3]{x+4} = \sqrt[3]{y^3}$$

$$\sqrt[3]{x+4} = y$$

$$\boxed{f^{-1}(x) = \sqrt[3]{x+4}}$$

$$\boxed{f^{-1}(x) = \pm\sqrt{x-3} + 2}$$

13) $f(x) = \frac{\sqrt{x-2}}{4} + 5$

$$y = \frac{\sqrt{x-2}}{4} + 5$$

$$x = \frac{\sqrt{y-2}}{4} + 5$$

$$-5 \quad -5$$

$$4(x-5) = \sqrt{y-2}$$

$$(4x-20)^2 = (\sqrt{y-2})^2$$

14) $f(x) = \sqrt{x-3} + 4$

$$y = \sqrt{x-3} + 4$$

$$x = \sqrt{y-3} + 4$$

$$-4 \quad -4$$

$$(x-4)^2 = (\sqrt{y-3})^2$$

$$(x-4)^2 = y-3$$

$$+3 \quad +3$$

$$(x-4)^2 + 3 = y$$

$$\boxed{f^{-1}(x) = (x-4)^2 + 3}$$

15) $f(x) = (4x-2)^2$

$$y = (4x-2)^2$$

$$\sqrt{y} = \sqrt{(4x-2)^2}$$

$$\pm\sqrt{y} = 4x-2$$

$$+2 \quad +2$$

$$\pm\sqrt{y} + 2 = 4x$$

$$\pm\sqrt{y} + \frac{2}{4} = y$$

$$\pm\sqrt{y} + \frac{1}{2} = y$$

$$\frac{1}{4} \sqrt{y} + \frac{1}{2} = y$$

Determine if the following functions are inverses of each other using $f(g(x))$ and $g(f(x))$.

16. $f(x) = \frac{2}{3}x - 12$ and $g(x) = \frac{3}{2}x + 18$ 17. $f(x) = x^2 - 4$ and $g(x) = \pm\sqrt{x+4}$

$f(g(x))$
 $\frac{2}{3}x - 12$
 $\frac{2}{3}(\frac{3}{2}x + 18) - 12$
 $x + 12 - 12$
 x ✓

$g(f(x))$
 $\frac{3}{2}x + 18$
 $\frac{3}{2}(\frac{2}{3}x - 12) + 18$
 $x - 18 + 18$
 x ✓

Yes

$f(g(x))$
 $x^2 - 4$
 $(\pm\sqrt{x+4})^2 - 4$
 $(\pm\sqrt{x+4})^2 - 4$
 $x + 4 - 4$
 x ✓

$g(f(x))$
 $\pm\sqrt{x+4}$
 $\pm\sqrt{x^2 - 4 + 4}$
 $\pm\sqrt{x^2 - 4 + 4}$
 $\pm\sqrt{x^2}$
 $\pm|x|$
 $\neq \pm\sqrt{x+4}$

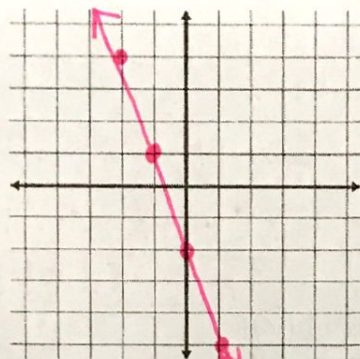
No

Find the inverse then graph the function and its inverse and tell if the function is one-to-one.

18. $f(x) = -3x - 2$

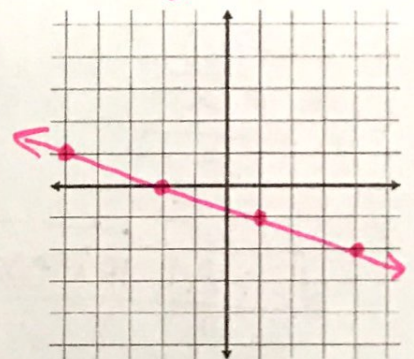
inverse: $f^{-1}(x) = \frac{-x-2}{3}$

x	y
-2	4
-1	1
0	-2
1	-5
2	-8



$y = -3x - 2$
 $x = -3y - 2$
 $x + 2 = -3y$

x	y
4	-2
1	-1
-2	0
-5	1
-8	2

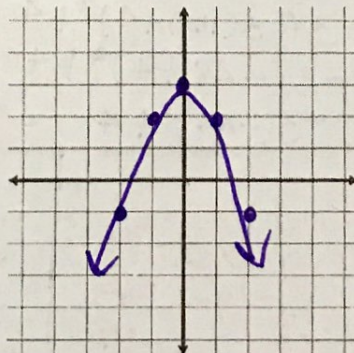


$-\frac{x}{3} - \frac{2}{3} = y$

19. $f(x) = -x^2 + 3$

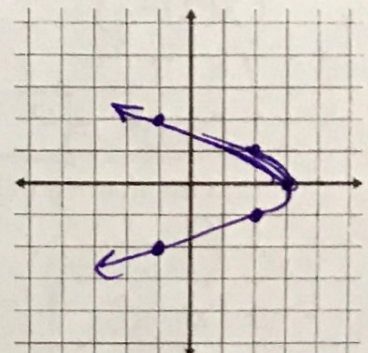
inverse: $f^{-1}(x) = \pm\sqrt{-x+3}$

x	y
-2	-1
-1	2
0	3
1	2
2	-1



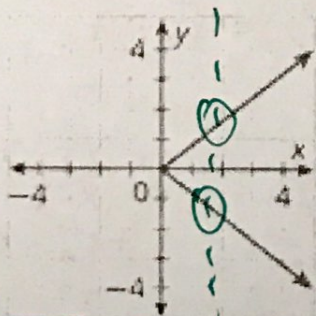
$y = -x^2 + 3$ $x - 3 = -y^2$
 $x = -y^2 + 3$ $\sqrt{-x+3} = |y|$

x	y
-1	-2
2	-1
3	0
2	1
-1	2



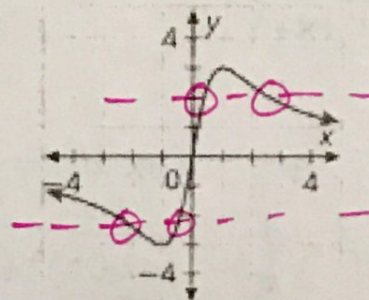
State whether the function is one-to-one.

20.



No, Does not pass vertical line test to be a function

21.



No, Does not pass the horizontal line test.