Solving and Writing Polynomials Given ALL Zeros

Complex Conjugate Root Theorem: If "a+bi" is a root of a polynomial, then "a-bi" is also a root.

Ex: $\frac{3x^2}{3} = \frac{-12}{3}$ $x = \pm 2i$

$\sqrt{x^2} = \sqrt{-4}$ $x = 2i, -2i$

Irrational Root Theorem: If "$a+\sqrt{b}$" is a root of a polynomial, where a is rational and $\sqrt{b}$ is irrational, then "$a-\sqrt{b}$" is also a root.

Ex: $\sqrt{(x-4)^2} = \sqrt{3}$

$x - \cancel{4} = \pm\sqrt{3}$ $x = 4 \pm \sqrt{3}$

$\cancel{+4} \quad +4$ $\boxed{x = 4+\sqrt{3}, 4-\sqrt{3}}$

Solve the following polynomial.

1. $x^4 + 4x^3 - x^2 + 16x - 20 = 0$

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -1 & 16 & -20 \\ & \downarrow & 1 & 5 & 4 & 20 \\ \hline & 1 & 5 & 4 & 20 & \boxed{0} \end{array}$$

$\boxed{x = -5, 2i, -2i, 1}$

$(x^3 + 5x^2) + (4x + 20) = 0$

$x^2(x+5) + 4(x+5) = 0$

$(x+5)(x^2+4) = 0$

$x^2 + \cancel{4} = 0$

$\cancel{-4} \quad -4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

2. $x^3 + 13x - 85 = 31$

Find the polynomial in standard form given the roots.

3. 1, -2, 2i and -2i

4. 2 + i, $\sqrt{3}$, 1 , 2 − i , $-\sqrt{3}$

$f(x) = (x-(2+i))(x-(2-i))(x-\sqrt{3})(x+\sqrt{3})(x-1)$

$f(x) = (x-2-i)(x-2+i)(x^2-\sqrt{9})(x-1)$

$f(x) = (x^2 - 2x + \cancel{xi} - 2x + 4 - \cancel{2i} - \cancel{xi} + \cancel{2i} - i^2)(x^2-3)(x-1)$

$f(x) = (x^2 - 4x + 4 - i^2)(x^3 - x^2 - 3x + 3)$

$f(x) = (x^2 - 4x + 4 + 1)(x^3 - x^2 - 3x + 3)$

$f(x) = (x^2 - 4x + 5)(x^3 - x^2 - 3x + 3)$

$f(x) = x^5 - x^4 - 3x^3 + 3x^2 - 4x^4 + 4x^3 + 12x^2 - 12x + 5x^3 - 5x^2 - 15x + 15$

$\boxed{f(x) = x^5 - 5x^4 + 6x^3 + 10x^2 - 27x + 15}$

5. 2i, 1 + $\sqrt{2}$, 3 , −2i , 1 − $\sqrt{2}$

6. -i, 2, 0 , i