

Solving Absolute Value Equations

Absolute Value Equations:

- Isolate the absolute value
- Set up 2 equations since what's inside the absolute value can be positive or negative.
- Solve both equations & check using the original equation.

Extraneous Solutions:

a solution for an equation derived from an original equation that is not a solution of the original equation.

Ex: Solve and check

1) $|2y - 4| = 12$

$$\begin{array}{l} 2y - 4 = 12 \\ 2y = 16 \\ \boxed{y = 8} \end{array} \quad \begin{array}{l} 2y - 4 = -12 \\ 2y = -8 \\ \boxed{y = -4} \end{array}$$

check: $|2(8) - 4| = 12$
 $|12| = 12$
 $12 = 12 \checkmark$
 $|2(-4) - 4| = 12$
 $|-12| = 12$
 $12 = 12 \checkmark$

2) $|3x + 2| = 7$

$$\begin{array}{l} 3x + 2 = 7 \\ 3x = 5 \\ \boxed{x = \frac{5}{3}} \end{array} \quad \begin{array}{l} 3x + 2 = -7 \\ 3x = -9 \\ \boxed{x = -3} \end{array}$$

3) $3|4w - 1| - 5 = 10$

$$\begin{array}{r} 3|5| - 5 \\ \hline 3|4w - 1| = 15 \\ \hline |4w - 1| = 5 \end{array}$$

$$\begin{array}{l} 4w - 1 = 5 \\ 4w = 6 \\ \boxed{w = \frac{3}{2}} \end{array} \quad \begin{array}{l} 4w - 1 = -5 \\ 4w = -4 \\ \boxed{w = -1} \end{array}$$

4) $2|3x - 1| + 5 = 33$

$$\begin{array}{l} 2|3x - 1| = 28 \\ |3x - 1| = 14 \\ 3x - 1 = 14 \\ 3x = 15 \\ \boxed{x = 5} \end{array} \quad \begin{array}{l} 3x - 1 = -14 \\ 3x = -13 \\ \boxed{x = -\frac{13}{3}} \end{array}$$

5) $|2x + 5| = 3x + 4$

$$\begin{array}{r} 2x + 5 = 3x + 4 \\ -2x \quad -2x \\ \hline 5 = x + 4 \\ -4 \quad -4 \\ \hline \boxed{1 = x} \end{array} \quad \begin{array}{r} 2x + 5 = -3x - 4 \\ +3x \quad +3x \\ \hline 5x + 5 = -4 \\ -5 \quad -5 \\ \hline 5x = -9 \\ x = -\frac{9}{5} \end{array}$$

check: $7 = 7 \checkmark$
 extraneous $1.4 = -1.4 \times$

6) $|2x + 3| = 3x + 2$

$$\begin{array}{l} 2x + 3 = 3x + 2 \\ 3 = x + 2 \\ \boxed{1 = x} \\ 2x + 3 = -3x - 2 \\ 5x + 3 = -2 \\ 5x = -5 \\ x = -1 \\ \text{extraneous} \end{array}$$