Rational Root Theorem

Rational root theorem: If P(x) has a rational root $\frac{p}{q}$, then p is a factor of the constant and q is a factor of the leading coefficient.

$\frac{p}{q}$ = factors of the constant / factors of leading coefficient

List all possible rational roots. Then find any actual roots (remember synthetic division can help us find roots!)

1) $x^3 + 3x^2 - 6x - 8 = 0$ (q: 1; p: ~~2,4,8,1~~) Possible roots: $\frac{P}{q} = \pm 2, \pm 4, \pm 8, \pm 1$

-2	1	3	-6	~~-8~~
	↓	-2	-2	16
	1	1	-8	8

2	1	3	-6	-8
	↓	2	10	8
	1	5	4	0

4, 1 / 4, 5

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x = -4, -1, 2$

2) $12x^3 + 31x^2 - 17x - 6 = 0$ (q: 2,6,3,4,1,12; p: ~~8,3,1,6~~) Possible roots: $\pm\frac{3}{2}, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{4}, \pm 3, \pm\frac{1}{4}, \pm\frac{1}{3}, \pm\frac{2}{3}$

Hmmm....that looks like A LOT of possibilities to try!! How can we narrow it down?

Remember a root is where the graph crosses the x-axis... $\pm 2, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm 6$

1) Enter in the polynomial
2) Graph
3) Use the graph to find a good guess for a factor—I think -3 looks like a winner! (Hint: you may want to check to make sure you are correct in your guess—(The y-value should be zero at your root!)
4) We can now use synthetic division and factor to find our other answers

-3	12	31	-17	-6
	↓	-36	15	6
	12	-5	-2	0

-24 / -8, 3 / -5

$12x^2 - 5x - 2 = 0$

$(12x^2 - 8x) + (3x - 2) = 0$

$4x(3x-2) + 1(3x-2) = 0$

$(3x-2)(4x+1) = 0$

5) Check your work using the graph! Did it cross the graph at your answers?

$x = \frac{2}{3}, -\frac{1}{4}, -3$

$3x - 2 = 0$ (+2 +2)

$3x = 2$

$x = \frac{2}{3}$

$4x + 1 = 0$ (-1 -1)

$4x = -1$

$x = -\frac{1}{4}$

List all possible rational roots. Then find all actual roots.

3) $2x^3 + 13x^2 + 17x - 12 = 0$ **4)** $x^4 - 5x^2 + 4 = 0$

2 1,2 2,6,4,3,1,12

$\frac{p}{q} = \pm \frac{2,6,4,3,1,12}{1,2} = \pm 2, \pm 1, \pm 6, \pm 3, \pm 4, \pm \frac{3}{2}, \pm \frac{1}{2}, \pm 12$

$-4 \mid 2 \quad 13 \quad 17 \quad -12$

$\quad\quad\downarrow \quad -8 \quad -20 \quad 12$

$\quad\quad 2 \quad 5 \quad -3 \quad \boxed{0}$

$2x^2 + 5x - 3 = 0$

$(x + \frac{6}{2})(x - \frac{1}{2}) = 0$

$(x+3)(2x-1) = 0$

$x = -3, \frac{1}{2}, -4$

-6

6 -1

5

5) $4x^3 - 12x^2 - x + 3 = 0$ **6)** $x^3 + x^2 - 7x + 2 = 0$

HW: #'s 11, 13, 14, 20, 22