Pascal's Triangle and The Binomial Theorem

Expand $(x + 2)^2$ $\quad (x+2)(x+2)$

x^2+4x+4

Now expand $(x + 2)^5$. Think it is going to take a long time???

Here is a short cut: Pascal's Triangle

1
1 1
1 2 1
#2 → 1 3 3 1
1 4 6 4 1
#1 → 1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

We will use this formula:

$(a + b)^n = \_\_\ a^n b^0 + \_\_\_\_\_\_\ a^{n-1} b^1 + \_\_\ a^{n-2} b^2 + \dots + \_\_\_\_\_\_\ a^0 b^n$

The blanks represent the numbers in the corresponding row of Pascal's Triangle.

Expand

1. $(x + 2)^5$

$1\ (x)^5\ (2)^0 = 1x^5(1) = 1x^5$
$5\ (x)^4\ (2)^1 = 5x^4(2) = 10x^4$
$10\ (x)^3\ (2)^2 = 10x^3(4) = 40x^3$
$10\ (x)^2\ (2)^3 = 10x^2(8) = 80x^2$
$5\ (x)^1\ (2)^4 = 5x(16) = 80x$
$1\ (x)^0\ (2)^5 = 1(1)(32) = 32$

$\boxed{x^5+10x^4+40x^3+80x^2+80x+32}$

2. $(2x + 4)^3$

$1\ (2x)^3\ (4)^0 = 1(8x^3)(1) = 8x^3$
$3\ (2x)^2\ (4)^1 = 3(4x^2)(4) = 48x^2$
$3\ (2x)^1\ (4)^2 = 3(2x)(16) = 96x$
$1\ (2x)^0\ (4)^3 = 1(1)(64) = 64$

$\boxed{8x^3+48x^2+96x+64}$

3. $(2x - 3)^5$

$1\ (2x)^5\ (-3)^0 = 1(32x^5)(1) = 32x^5$
$5\ (2x)^4\ (-3)^1 = 5(16x^4)(-3) = -240x^4$
$10\ (2x)^3\ (-3)^2 = 10(8x^3)(9) = 720x^3$
$10\ (2x)^2\ (-3)^3 = 10(4x^2)(-27) = -1080x^2$
$5\ (2x)^1\ (-3)^4 = 5(2x)(81) = 810x$
$1\ (2x)^0\ (-3)^5 = 1(1)(-243) = -243$

$\boxed{32x^5-240x^4+720x^3-1080x^2+810x-243}$