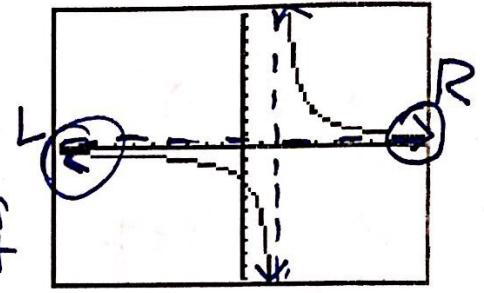


Part I: Let's consider the curve  $f(x) = \frac{5}{x-2}$  and learn about some of the characteristics of the function.



1. Use interval notation to describe the domain of  $f(x)$ . X-values (Look left to right)  
 $(-\infty, 2) \cup (2, \infty)$  OR All  $\mathbb{R}$ 's except  $x \neq 2$

2. Use interval notation to describe the range of  $f(x)$ . Y-values (Look Bottom to Top)  
 $(-\infty, 0) \cup (0, \infty)$  OR All  $\mathbb{R}$ 's except  $y \neq 0$

3. What are the roots or zeros of  $f(x)$ . (x-intercepts) NONE

4. Describe the end behavior of  $f(x)$ .  $x \rightarrow -\infty, f(x) \rightarrow 0$   $x \rightarrow \infty, f(x) \rightarrow 0$

5. When will  $f(x)$  intersect the y-axis? (0, -2.5)  
 (plug zero in for x)  
 $\frac{5}{0-2} = -\frac{5}{2} = -2.5$

Part 2: Find the roots and y-intercept of the functions.

1.  $f(x) = \frac{-2x+5}{x+1}$

Top  
 Plug in  
 Roots  $\frac{5}{2}$  or 2.5  
 y-intercept (0, 5)

$$\begin{aligned} -2x+5 &= 0 & \frac{5}{1} &= 5 \\ -2x &= -5 & & \\ x &= \frac{5}{2} & & \end{aligned}$$

2.  $f(x) = \frac{3x-2}{x}$

Roots  $\frac{2}{3}$   
 y-intercept NONE

$$\begin{aligned} 3x-2 &= 0 & \frac{-2}{0} &= \text{undef.} \\ 3x &= 2 & & \\ x &= \frac{2}{3} & & \end{aligned}$$

3.  $f(x) = \frac{x^2-6x+8}{x^2-16}$   $\frac{(x-4)(x-2)}{(x+4)(x-4)}$

Roots 2  
 y-intercept  $(0, -\frac{1}{2}) = \frac{x-2}{x+4}$

$$\frac{8}{-16} = -\frac{1}{2} \quad \frac{-2}{4} = -\frac{1}{2}$$

4.  $f(x) = \frac{2x^2-6x-8}{x^2-4}$

Roots 4, -1  
 y-intercept (0, 2)

$$\frac{2(x^2-3x-4)}{(x+2)(x-2)} = \frac{2(x-4)(x+1)}{(x+2)(x-2)}$$

5.  $f(x) = \frac{x^2-6x+8}{x^2-9}$

Roots \_\_\_\_\_  
 y-intercept \_\_\_\_\_

6.  $f(x) = \frac{(x^3-3x^2)-(x+3)}{x^2-16}$

Roots 3, 1, -1  
 y-intercept  $(0, -\frac{3}{16})$

$$\begin{aligned} &x^2(x-3) - 1(x-3) \\ &(x-3)(x^2-1) \\ &\frac{(x-3)(x+1)(x-1)}{(x+4)(x-4)} \end{aligned}$$

## Rational Functions

Asymptotes	
Vertical (VA)	Horizontal (HA)
<p>* occurs when the function is undefined.</p> <p>* Find VA by setting the denominator equal to zero &amp; solving for x.</p>	<p>* The graph will have one OR NO HA, by comparing the highest degree of the numerator &amp; denominator.</p> <p>① Bigger Degree on Top: NO HA</p> <p>② Bigger Degree on Bottom: <math>y=0</math></p> <p>③ Degrees are same:  <math>y = \frac{\text{lead coefficient on top}}{\text{lead coefficient on bottom}}</math></p>

Ex: Identify the zeros, the vertical and horizontal asymptotes.

1.  $f(x) = \frac{2x^1 - 4}{x^1 - 1}$

$$f(x) = \frac{2(x-2)}{x-1}$$

Top Zeros: 2

y-intercept:  $\frac{-4}{-1} = 4$  (0, 4)

Bottom VA:  $x=1$

deg. HA:  $y = \frac{2}{1} = 2$

3.  $f(x) = \frac{x+4}{2x^2-18}$

$$f(x) = \frac{x+4}{2(x^2-9)}$$

$$f(x) = \frac{x+4}{2(x+3)(x-3)}$$

T Zeros: -4

y-intercept:  $\frac{-4}{18} = \frac{-2}{9}$  (0,  $-\frac{2}{9}$ )

B VA:  $x=3$  &  $x=3$

D HA:  $y=0$

2.  $f(x) = \frac{x^2-2x}{x^2-2x-3}$

$$f(x) = \frac{x(x-2)}{(x-3)(x+1)}$$

T Zeros: 0 & 2

y-intercept:  $\frac{0}{-3} = 0$  (0, 0)

B VA:  $x=3$  &  $x=-1$

D HA:  $y=1$

4.  $f(x) = \frac{x^3-x^2-6x}{-4x^2+4x+8}$

$$f(x) = \frac{x(x^2-x-6)}{-4(x^2-x-2)}$$

$$f(x) = \frac{x(x-3)(x+2)}{-4(x-2)(x+1)}$$

Zeros: 0, 3, -2

y-intercept: (0, 0)

VA:  $x=2$  &  $x=-1$

HA: NONE