Graphing Rational Functions with Slant Asymptotes and Holes Notes

Holes: An open Circle on the graph, (x,y). This is where there is a break in the graph and occurs when there is a Common factor between the numerator and denominator. The X-coordinate is found by setting the common factor equal to zero. The Y-coordinate is found by substituting the x-coordinate into the simplified equation.

$\boxed{\text{hole: } (3, \tfrac{2}{5})}$

x: $x-3=0$, $x=3$ y: $\frac{3-1}{3+2}=\frac{2}{5}$

Ex: $f(x)=\dfrac{x^2-4x+3}{x^2-x-6} = \dfrac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+2)} = \dfrac{x-1}{x+2}$

Slant Asymptotes: If the degree on the top is one degree higher than the degree on the bottom, then the function has a slant asymptote, $y=mx+b$.

- Use long division to find the equation.

Ex: $f(x)=\dfrac{x^2+x-6}{x+2}$

$$\begin{array}{r} x-1 \\ x+2 \overline{) \cancel{x^2}+x-6} \\ -\cancel{x^2}+2x \\ \hline -1x-6 \end{array}$$

$\boxed{\text{SA: } y=x-1}$

1.) $f(x)=\dfrac{x^2-6x+8}{x^2-16} = \dfrac{\cancel{(x-4)}(x-2)}{(x+4)\cancel{(x-4)}} = \dfrac{x-2}{x+4}$

x-intercept(s): (2,0)

y-intercepts: (0, -½)

VA: X= -4

HA: Y= 1

SA: NONE

hole: (4, ¼)

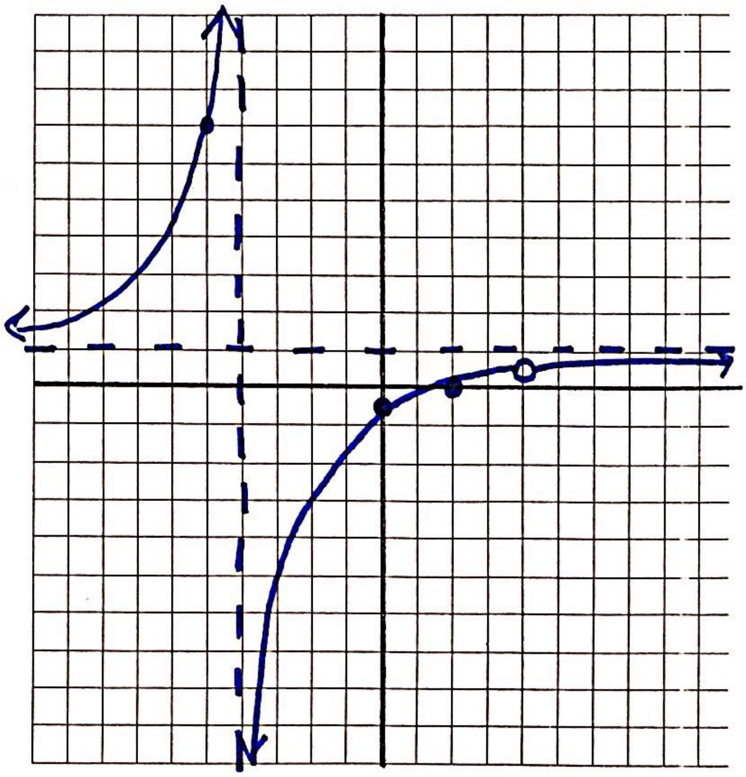
domain: All IR's except x ≠ -4, 4

range: All IR's except y ≠ 1, ¼

End Behavior:

$as\, x \to \infty, f(x) \to$: 1

$as\, x \to -\infty, f(x) \to$: 1



$\dfrac{4-2}{4+4} = \dfrac{2}{8} = \dfrac{1}{4}$

Test: $\dfrac{-5-2}{-5+4} = \dfrac{-7}{-1} = 7$

(-5, 7)

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2.) f(x) $= \dfrac{x^2 - x - 12}{x} = \dfrac{(x-4)(x+3)}{x}$

x-intercept(s): (4,0) (-3,0)

y-intercept: NONE

VA: X=0

HA: NONE

SA: y= X-1

hole: NONE

domain: All IR's except x≠0

range: All IR's

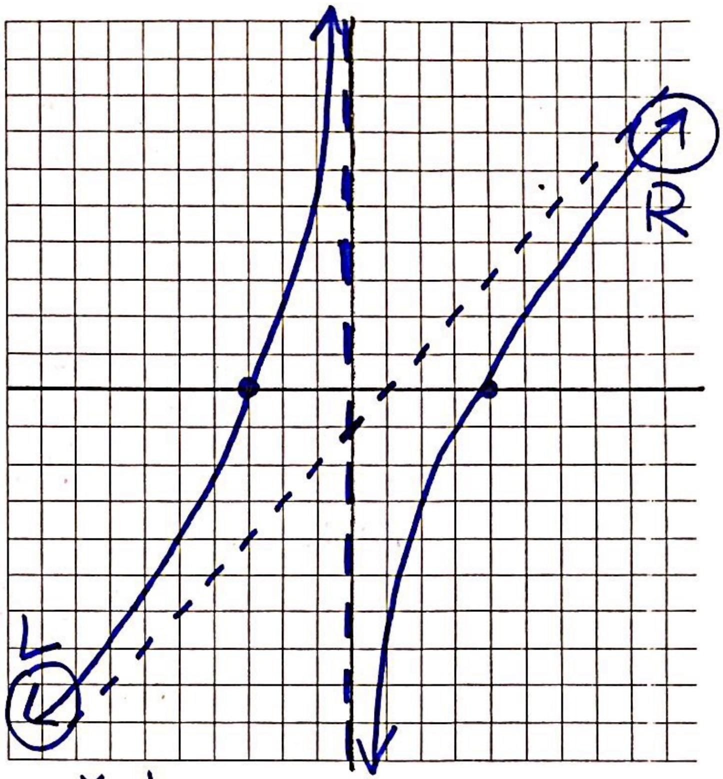
End Behavior:

$as\ x \to \infty, f(x) \to$: ∞

R

$as\ x \to -\infty, f(x) \to$: $-\infty$

L



$$\begin{array}{r} x-1 \\ x\overline{)\,x^2 - x - 12} \\ \underline{-x^2} \\ -x-12 \end{array}$$

3.) f(x) = $\dfrac{x^2 - 4x + 3}{x^2 - x - 6}$

x-intercept(s): ____

y-intercept: ____

VA: ____

HA: ____

SA: ____

hole: ____

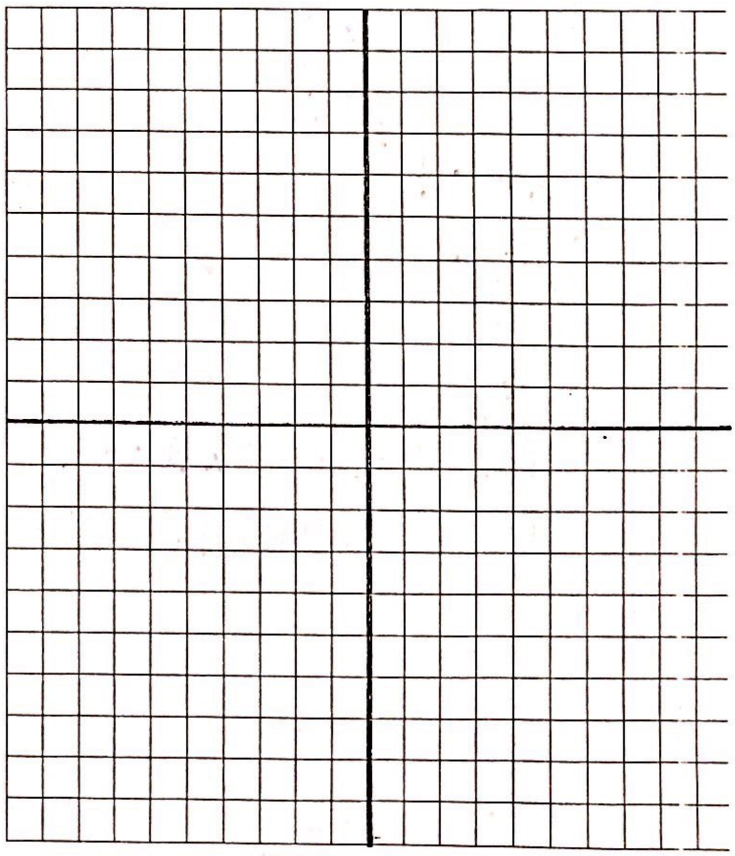
domain: ____

range : ____

End Behavior:

$as\ x \to \infty, f(x) \to$: ____

$as\ x \to -\infty, f(x) \to$: ____



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