Finding Inverses of f(x)

- $f(x)$ and y are the Same thing.
- $f^{-1}(x)$ is the notation for inverse of a function.

5 things to know about inverses:

1) Inverse functions undo each other.
2) Inverse functions switch x & y. $(x, y) \longrightarrow (y, x)$
3) The original & inverse graphs are symmetric across $y = x$.
4) To find an inverse, switch x & y, & solve for y.
5) Use $f^{-1}(x)$ to express the inverse.

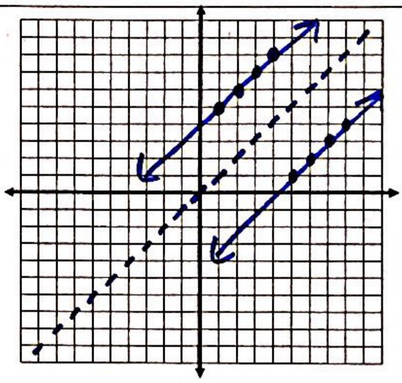
How the inverse looks graphically

$f(x) = x + 4$

x	y
1	5
2	6
3	7
4	8

$f^{-1}(x) = x - 4$

x	y
5	1
6	2
7	3
8	4



Steps to find an inverse:

1. Replace f(x) with y
2. Switch x & y
3. Solve for y
4. Replace y with $f^{-1}(x)$.

Ex: Find the inverse.

1. $f(x) = 2x + 12$

$$y = 2x + 12$$
$$x = 2y + 12$$
$$\frac{x}{2} - \frac{12}{2} = \frac{\cancel{2}y}{\cancel{2}}$$
$$\frac{x}{2} - 6 = y$$
$$\boxed{f^{-1}(x) = \frac{x}{2} - 6}$$

2. $f(x) = 3x^2 - 5$

$$x = 3y^2 - 5$$
$$\frac{x+5}{3} = \frac{\cancel{3}y^2}{\cancel{3}}$$
$$\sqrt{\frac{x+5}{3}} = \sqrt{y^2}$$
$$\pm\sqrt{\frac{x+5}{3}} = y$$
$$\boxed{f^{-1}(x) = \pm\sqrt{\frac{x+5}{3}}}$$

3. $f(x) = (4x+9)^2 - 10$

$x = (4y+9)^2 - 10$

$\sqrt{x+10} = \sqrt{(4y+9)^{\cancel{2}}}$

$\pm\sqrt{x+10} = 4y+9$

$\frac{\pm\sqrt{x+10} - 9}{4} = \frac{4y}{4}$

$\frac{\pm\sqrt{x+10} - 9}{4} = y$

$\boxed{f^{-1}(x) = \frac{\pm\sqrt{x+10} - 9}{4}}$

4. $f(x) = \sqrt[3]{\frac{3x-4}{12}}$

$(x)^3 = \left(\sqrt[\cancel{3}]{\frac{3y-4}{12}}\right)^{\cancel{3}}$

$12(x^3) = \left(\frac{3y-4}{\cancel{12}}\right)\cancel{12}$

$12x^3 = 3y - 4$

$\frac{12x^3}{3} + \frac{4}{3} = \frac{\cancel{3}y}{\cancel{3}}$

$4x^3 + \frac{4}{3} = y$

$\boxed{f^{-1}(x) = 4x^3 + \frac{4}{3}}$

Ex: Verify that f(x) and g(x) are inverse functions of each other using f(g(x)) and g(f(x)).

5. f(x) = $\frac{2}{3}x + 6$ and g(x) = $\frac{3}{2}x - 9$

$f(g(x))$

$\frac{2}{3}x + 6$

$\frac{2}{3}\left(\frac{3}{2}x - 9\right) + 6$

$x - 6 + 6$

x ✓

$g(f(x))$

$\frac{3}{2}x - 9$

$\frac{3}{2}\left(\frac{2}{3}x + 6\right) - 9$

$x + 9 - 9$

x ✓

$\boxed{\text{Yes}}$

6. f(x) = $\sqrt{3x}$ and g(x) = $\frac{x^2}{3}$ for x ≥ 0