Name:____________________ Date: ____________

**Margin of Error:**

- gives a margin on how much the response of a sample would differ from the responses of a population

z-score Standard Deviation

$$M.E. = \pm(z^*)\left(\frac{\sigma}{\sqrt{n}}\right)$$

sample size

- if the percent of the sample responding is $\bar{X}$, then the percent of the **population** that would respond the same way is between:

$$\bar{x} - M.E. \text{ and } \bar{x} + M.E.$$

**Ex 1** Imagine a polling organization that surveys 1,000 voters in a city to find out how they plan to vote in the upcoming election. The organization reports that 58% of the city plans to vote for Smith, and that the survey has a margin of error of $\pm 3\%$.

n

a. Find the interval in which the population percent is most likely to lie.

58% ± 3% = [55%, 61%]

**Confidence Intervals:** Based on the Empirical Rule (68-95-99.7), you need a z-score associated to calculate the confidence interval.

$$\bar{x} \pm (z^*)\left(\frac{\sigma}{\sqrt{n}}\right)$$

Mean ± ME

| Level of Confidence | Z-Score |
|---|---|
| 90% | 1.645 |
| 95% | 1.96 |
| 98% | 2.33 |
| 99% | 2.575 |

**Ex 2.** A company that produces white bread is concerned about the distribution of the amount of sodium in its bread. The company takes a simple random sample of 100 slices of bread and computes the sample mean to be 103 milligrams of sodium per slice and the standard deviations to be 10 milligrams. Given a 99% confidence, find:

a. the **margin of error** $ME = \pm(z^*)\left(\frac{\sigma}{\sqrt{n}}\right) = \pm(2.575)\left(\frac{10}{\sqrt{100}}\right) = \pm 2.575$

b. a **confidence interval** for estimating the population mean

$\bar{X} \pm ME = 103 \pm 2.575 = [100.425, 105.575]$

**Ex 3.** A sample of 100 observations is collected and yields a mean of 75 and a standard deviation of 8. Find a 95% confidence interval for the true population average.

n= 100 $\bar{x}$= 75 σ= 8 Confidence Level= 1.96

$\bar{X} \pm ME = 75 \pm 1.96\left(\frac{8}{\sqrt{100}}\right) = 75 \pm 1.568$

$\boxed{[73.432, 76.568]}$

**Ex 4.** SAT scores are normally distributed with a mean of 1000 and a standard deviation of 120. Mr. Markley's calculus students earned the scores shown below. $\bar{x}=1000 \quad \sigma=120$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 940 | 1040 | 1170 | 1120 | 1070 | 990 | 1080 | 1000 |
| 1140 | 850 | 1330 | 990 | 1220 | 1120 | 1190 | 1100 |

a. Find the 99% confidence interval estimate of the mean.

$$\bar{x} \pm z^*\left(\frac{\sigma}{\sqrt{n}}\right) = 1000 \pm 2.575\left(\frac{120}{\sqrt{16}}\right) = 1000 \pm 77.25 \quad \boxed{[922.75, 1077.25]}$$

b. What conclusions can be drawn concerning the SAT's mean score and the calculus class' mean score?

99% confident that the calculus class will score anywhere between the interval, so it's not necessarily different than the true average of all SAT takers.

**Ex 5.** Apartment rental rates. You want to rent an unfurnished one-bedroom apartment for next semester. The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is $540. Assume that the standard deviation is $80.

a. Find a 95% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community.

$\bar{x}=540 \quad \sigma=80 \quad n=10$

b. Compare the margin of error for intervals with 90, 95, and 99% confidence:

90%: $540 \pm (1.645)\left(\frac{80}{\sqrt{10}}\right)$ $[498.38, 581.62]$

95%: $540 \pm (1.96)\left(\frac{80}{\sqrt{10}}\right)$ $[490.42, 589.58]$

99%: $540 \pm (2.575)\left(\frac{80}{\sqrt{10}}\right)$ $[474.86, 605.14]$

$ME = z^*\left(\frac{\sigma}{\sqrt{n}}\right)$

As the sample size (n) **increases**, the margin of error (m) decreases.

As the confidence level (C) **increases**, the margin of error (m) increases.

z-score

As the standard deviation ($\sigma$) **increases**, the margin of error (m) increases.

**Ex 6.** An economist wants to estimate the first year's mean income of college graduates who have had the profound wisdom to take a statics course. How many such incomes must be found if we want to be 95% confident that the sample mean is within $500 of the true population mean? Assume that a previous study of such incomes has shown a $\sigma = \$6250$.

$$ME = z^*\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$500 = 1.96\left(\frac{6250}{\sqrt{n}}\right)$$

$$\sqrt{n}(255.1) = \left(\frac{6250}{\cancel{\sqrt{n}}}\right)\cancel{\sqrt{n}}$$

$$255.1\sqrt{n} = 6250$$

$$(\sqrt{n})^2 = (24.5)^2$$

$$n = 600.25$$

$\boxed{600 \text{ incomes}}$