Algebra II Name________________

Solving Equations by Completing the Square Period____Date________

Another way to solve a quadratic equation is to "**complete the square**". First of all, think about how to square binomials (using FOIL):

$(x+5)^2 =$ $x^2+10x+25$ $(x+5)(x+5) = x^2+5x+5x+25$

$(x-8)^2 =$ $x^2-16x+64$ $2(x)(-8)$

$(7x-4)^2 =$ $49x^2-56x+16$ $2(7x)(-4)$

Now, looking at these problems, describe a pattern to quickly how to get each term in $(a+b)^2$.

$a^2+2ab+b^2$

Let's reverse your thinking: Fill in the blanks to make each of these true statements. $\left(\frac{b}{2}\right)^2$

$\left(\frac{18}{2}\right)^2$ x^2+18x+ 81 $= (x+$ 9 $)^2$ $(x+9)(x+9)$

x^2-20x+ ______ $= (x-$ ______ $)^2$

$\left(\frac{-4}{2}\right)^2$ x^2-4x+ 4 $= ($ $x-2$ $)^2$

x^2+5x+ $\frac{25}{4}$ $= ($ $x+\frac{5}{2}$ $)^2$ $\left(\frac{5}{2}\right)^2$

$x^2+2.6x+$ ______ $= ($ ________ $)^2$

$x^2-2\sqrt{5}\,x+$ ______ $= ($ ________ $)^2$

$3x^2+42x+$ ______ $= 3(x^2+$ ________ $) = 3(x+$ _____ $)^2$

So, if we wanted to solve $x^2-8x+15 = 0$, we could "complete the square" and say:
First, isolate your variables and force the leading coefficient to be 1. We'd get:

$x^2-8x = -15$ Now, add enough to both sides to "Complete the Square".

$x^2-8x+16 = -15+16$ Factor the left side, and simplify the other.

$(x-4)^2 = 1$ Take the square root of both sides.

$x-4 = \pm 1$ The ± sign is very important! Why? Now solve for x.

$$x = \pm 1 + 4 = \begin{Bmatrix} 1+4 \\ -1+4 \end{Bmatrix} \Rightarrow \boxed{x = 5 \ \ or \ \ x = 3}$$

Solve each of these equations by "Completing the Square". (Some are easier than others!)

1.) $\sqrt{(x-5)^2} = \sqrt{49}$

$x-5 = \pm 7$

$x-5 = 7$ $x-5 = -7$

$\boxed{x = 12 \quad x = -2}$

2.) $(5x-2)^2 = 4$

3.) $x^2+2x+1 = 25$

4.) $x^2+6x+9 = 100$

9, 3, 3, 6

$(x+3)(x+3) = 100$

$\sqrt{(x+3)^2} = \sqrt{100}$

$x+3 = \pm 10$

$x+3 = 10$

$\boxed{x = 7}$

$x+3 = -10$

$\boxed{x = -13}$

5.) $x^2 - 14x - 32 = 0$

$\left(\frac{-14}{2}\right)^2$ $\quad x^2 - 14x = 32$

$x^2 - 14x + \underline{49} = 32 + \underline{49}$

$(-7)^2$ $\quad \sqrt{(x-7)^2} = \sqrt{81}$

49 $\quad x - 7 = \pm 9$

$\boxed{x = 16, -2}$

6.) $y^2 + 12y + 35 = 0$

7.) $p^2 + 20p = -75$

8.) $K^2 - 80 = 16K$

$\left(-\frac{16}{2}\right)^2$ $\quad k^2 - 16k = 80$

$k^2 - 16k + \underline{64} = 80 + \underline{64}$

$\sqrt{(k-8)^2} = \sqrt{144}$

$k - 8 = \pm 12$

$\boxed{k = 20, -4}$

9.) $x^2 = 50 - 5x$

10.) $M^2 + M - 20 = 400$

$\left(\frac{1}{2}\right)^2$ $\quad m^2 + m = 420$

$m^2 + m + \frac{1}{4} = 420 + \frac{1}{4}$

$\frac{1}{4}$ $\quad \sqrt{\left(m + \frac{1}{2}\right)^2} = \sqrt{\frac{1681}{4}}$

$m + \frac{1}{2} = \pm \frac{41}{2}$

$\boxed{m = 20, -21}$

Sometimes the answers may not be rational...

11.) $x^2 - 8x = 30$

12.) $Y^2 = 9Y - 5$

$\left(-\frac{9}{2}\right)^2$ $\quad y^2 - 9y = -5$

$y^2 - 9y + \underline{81/4} = -5 + \underline{\frac{81}{4}}$

$\frac{81}{4}$ $\quad \sqrt{\left(y - \frac{9}{2}\right)^2} = \sqrt{\frac{61}{4}}$

$y - \frac{9}{2} = \pm \frac{\sqrt{61}}{2}$

$\boxed{y = \frac{9 \pm \sqrt{61}}{2} \text{ OR } y = \frac{9}{2} \pm \frac{\sqrt{61}}{2}}$

Sometimes the answers aren't even real...

13.) $z^2 + 8z = -20$

$\left(\frac{8}{2}\right)^2$ $\quad z^2 + 8z + \underline{16} = -20 + \underline{16}$

$(4)^2$ $\quad \sqrt{(z+4)^2} = \sqrt{-4}$

16 $\quad z + 4 = \pm 2i$

$\boxed{z = -4 \pm 2i}$

14.) $6x - x^2 = 48$