Arithmetic and Geometric Series

Series: the expression for the sum of the terms of a sequence.

Arithmetic Series: a series whose terms form an arithmetic sequence.

Geometric Series: a series whose terms form a geometric sequence.

Finite Series: have terms you can count individually from 1 to a final whole number.

Formula of an Arithmetic Series:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

n = # of terms, a_1 = 1st term, a_n = nth term

Sum of a Finite Geometric Series:

$$S_n = \frac{a_1(1-(r)^n)}{1-r}$$

r = common ratio

Evaluate the series to the given term.

1) 1+5 + 9 +... +77 (20 terms)

$n=20$

$a_1=1$

$a_n=77$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{20} = \frac{20}{2}(1+77)$$

$$\boxed{S_{20} = 780}$$

2) 3 – 2 – 7 –..., -72 (16 terms)

3) 4 + 12 + 36 + ...;S_6

4) 1 – 3 + 9 – 27 + ...; S_8

$a_1=1$

$r=-3$

$n=8$

$$S_n = \frac{a_1(1-(r)^n)}{1-r}$$

$$S_8 = \frac{1(1-(-3)^8)}{1-(-3)}$$

$$\boxed{S_8 = -1640}$$

Geo.

5) $\sum_{k=1}^{5}\left(\frac{1}{3}\right)^{k-1}$

$a_n = a_1 \cdot r^{n-1}$

$\sum_{k=1}^{5} 1\left(\frac{1}{3}\right)^{k-1}$

$a_1 \quad r$

$S_n = \frac{a_1(1-(r)^n)}{1-r}$

$S_n = \frac{1(1-(\frac{1}{3})^5)}{1-(1/3)} = \boxed{\frac{121}{81}}$

Arith.

6) $\sum_{n=1}^{30}(2n-1)$

$a_1 = 2(1)-1$

$a_1 = 1$

$a_{30} = 2(30)-1$

$a_{30} = 59$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{30} = \frac{30}{2}(1+59)$

$\boxed{S_{30} = 900}$

Find S_n for each arithmetic series.

7) $a_1 = 91, d = -4, a_n = 15$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{20} = \frac{20}{2}(91+15)$

$\boxed{S_{20} = 1060}$

$a_n = a_1 + (n-1)d$

$15 = 91 + (n-1)(-4)$

$15 = 91 - 4n + 4$

$15 = 95 - 4n$

$-80 = -4n$

$20 = n$

8) $d = 5, n = 16, a_n = 72$

$72 = a_1 + (16-1)5$

$72 = a_1 + 75$

$-3 = a_1$

$S_{16} = \frac{16}{2}(-3+72)$

$\boxed{S_{16} = 552}$

Find the first three terms of each arithmetic series.

9) $a_1 = 7, a_n = 139, S_n = 876$

$a_n = a_1 + (n-1)d$

$139 = 7 + (12-1)d$

$139 = 7 + 11d$

$132 = 11d$

$12 = d$

$S_n = \frac{n}{2}(a_1 + a_n)$

$876 = \frac{n}{2}(7+139)$

$2 \cdot 876 = \frac{n}{2}(146) \cdot 2$

$1752 = 146n$

$12 = n$

$\boxed{7, 19, 31}$

10) $n = 14, a_n = 53, S_n = 378$

Find the sum of each geometric series.

11) $a_1 = 625, a_5 = 81, r = \frac{3}{5}$

$S_n = \frac{a_1(1-(r)^n)}{1-r}$

$S_5 = \frac{625(1-(\frac{3}{5})^5)}{1-(3/5)} = \boxed{1441}$

12) $a_1 = 256, r = 0.75, n = 9$

Find a_1 for each geometric series.

13) $S_n = 244, r = -3, n = 5$

$244 = \frac{a_1(1-(-3)^5)}{1-(-3)}$

$4(244) = \frac{a_1(244)}{\cancel{4}} \cdot \cancel{4}$

$976 = a_1(244)$

$\boxed{4 = a_1}$

14) $S_n = 1022, r = 2, n = 9$