

Arithmetic and Geometric Sequences

In an arithmetic sequence, the difference between consecutive terms is constant. This difference is called the common difference.

In a geometric sequence, the ratio between consecutive terms is constant. This ratio is called the common ratio.

Ex: Is the sequence arithmetic, geometric or neither? If so, identify the common difference or common ratio.

1. 2, 5, 7, 12, ...

$\begin{matrix} \vee & \vee \\ 3 & 2 \end{matrix}$

Neither

2. 48, 45, 42, 39, ...

$d = -3$
Arithmetic

3. 7, 10, 13, 16, ...

$d = 3$
Arithmetic

4. 6, -24, 96, -384, ...

$r = -4$
Geometric

5. 8, 20, 32, 44, ...

$d = 12$
Arithmetic

6. 1, -6, 36, -216, ...

$r = -6$
Geometric

Arithmetic Formulas:

Recursive Formula

$$a_1 = \#$$

$$a_n = a_{n-1} + d$$

Explicit (Closed) Formula

$$a_n = a_1 + (n-1)d$$

$a_1 = 1^{\text{st}}$ term

$a_n =$ specific term

$n =$ term #

$d =$ common difference

$r =$ common ratio

Geometric Formulas:

Recursive Formula

$$a_1 = \#$$

$$a_n = a_{n-1} \cdot r$$

$$a_n = r a_{n-1}$$

Explicit (Closed) Formula

$$a_n = a_1 \cdot r^{n-1}$$

7. Find the common difference, the recursive formula, and the tenth term. **Arithmetic**

3, 9, 15, 21, ...

$d = 6$

Recursive:

$a_1 = 3$
 $a_n = a_{n-1} + 6$

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 3 + (10-1)6$$

$a_{10} = 57$

8. Write the explicit rule and the recursive rule. **Geometric**

$a_1 = 4$ and $r = 0.2$

Exp: $a_n = a_1 r^{n-1}$
 $a_n = 4(0.2)^{n-1}$

Rec: $a_1 = 4$
 $a_n = 0.2a_{n-1}$

9. Find the common difference, the recursive formula, and the tenth term.
5, 11, 17, 23 ...

10. Use the recursive formula to find the 6th term of the sequence. **Arithmetic**

$a_1 = 5$

$a_n = a_{n-1} + 4$

$d = 4$

$a_n = a_1 + (n-1)d$

$a_6 = 5 + (6-1)(4)$

$a_6 = 25$

11. Write an explicit rule and a recursive rule for the geometric sequence.

3, 6, 12, 24, ...

Exp: $a_n = 3(2)^{n-1}$

Rec: $a_1 = 3$
 $a_n = 2a_{n-1}$

12. Find the common ratio, the explicit formula, and the seventh term. **Geometric**

3, 1.5, 0.75, 0.375, ...

$r = 0.5$

Exp: $a_n = 3(0.5)^{n-1}$

$a_7 = 3(0.5)^{7-1}$

$a_7 = 0.046875$

13. Find the 19th term of the geometric sequence.

11, 33, 99, 297, ...

$a_n = a_1 r^{n-1}$
 $a_{19} = 11(3)^{19-1}$

$a_{19} = 4,261,625,379$

14. Use the recursive formula to find the 1st term and the 10th term of the sequence.

$a_6 = 2$

$a_n = a_{n-1} - 3$

$d = -3$

$a_1 = 17$

$a_2 = 14$

$a_3 = 11$

$a_4 = 8$

$a_5 = 5$

$a_6 = 2$ *start here

$a_7 = -1$

$a_8 = -4$

$a_9 = -7$

$a_{10} = -10$